tributed about $\hat{P}_o$ yields:

$$P_o - d \leq \hat{P}_o \leq P_o + d \tag{12}$$

where

$$d = Z\alpha/2[P_o(1 - P_o)/n]^{1/2}$$

and $Z\alpha$ refers to the number of standard deviations in the interval from the mean to $\alpha/2$ (for a 95% probability a normal probability table yields 1.96). For a preselected value of α and a value of P_o from one or more Monte Carlo simulations, the value of confidence limit (d) can be computed using Eq. (12). The computed confidence interval is of the same order of magnitude as $P(TF/SF)$ for large n.

Conclusions

The Monte Carlo simulation of peak loads, tumbling and the determination of the probabilities of success has been achieved through the use of analytical and empirical techniques. This simulation technique can be modified or improved to evaluate the design of CM as better and more refined information becomes available. Since the peak load analysis generates impact loading close to experimental values it can be concluded that the Monte Carlo simulation has adequately simulated land-landing.

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Inversion of Spin-Stabilized Spacecraft by Mass Translation—Some Practical Aspects

NORMAN H. BEACHLEY*
University of Wisconsin, Madison, Wis.

INVERSION can be a very useful maneuver for certain types of spin-stabilized spacecraft; e.g., for a satellite whose spin axis is kept parallel to that of the Earth, inversion twice a year would allow the same end to remain in the shade all year round, protecting devices mounted on the spin axis which are sensitive to solar radiation.

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* Assistant Professor of Mechanical Engineering.

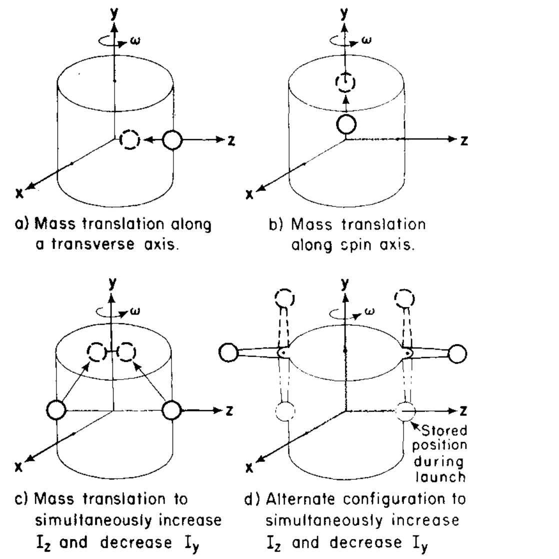

Fig. 1 Various configurations for a satellite inversion system (solid outline of control mass is position in normal operation).

The most straightforward method of inverting a spin-stabilized satellite is by applying a torque electromagnetically or by mass expulsion, but the large angular impulse required to turn the angular momentum vector through 180° makes this approach unattractive if inversion is to be done on a regular basis. For example, it would take about 5 lb of H_2O_2 (Specific Impulse = 160 sec) to invert a spin-stabilized satellite having a moment of inertia of 80 slug-ft^2 about the spin axis, a spin rate of 100 rpm, and a $25\frac{1}{4}$ in. radius for the jets. (System inefficiencies are ignored.)

An alternate approach is to allow the momentum vector to remain fixed in inertial space, but to cause the satellite to invert itself by proper manipulation of its moments of inertia.[1] With this technique, the spacecraft ends up rotating in the opposite direction than if precession were used, but this should normally be of no consequence.

If a rigid body rotates about its axis of maximum or minimum moment of inertia, the rotation is stable. On the other hand, if it rotates about the principal axis of intermediate moment of inertia, the rotation is unstable. In view of this, inversion may be accomplished in the following manner: with the satellite spin-stabilized about **y**, its axis of maximum moment of inertia, a mass is moved in a manner that causes 1) a slight rotation of principal axes **x**, **y**, and **z** with respect to the spacecraft so that **y** no longer coincides with the momentum vector, being offset from it by the angle α_o, and 2) a change in ratios of the principal moments of inertia so that **y** is now the principal axis of intermediate moment of inertia, with I_z just slightly greater than I_y. The unstable spacecraft will then proceed to invert itself. When it has tilted as far as it is going to (almost 180°), the control mass is returned to its original position, and the inverted spacecraft will again be spinning in a stable manner. The small amount of nutation caused by the maneuver can be readily eliminated by a nutation damper.

If we define the inversion maneuver as beginning at the completion of the control mass movement, it is completely described by Euler's dynamical equations for a rigid body,

$$I_x d\omega_x/dt + (I_y - I_z)\omega_y\omega_z = 0$$

$$I_y d\omega_y/dt + (I_z - I_x)\omega_z\omega_x = 0$$

$$I_z d\omega_z/dt + (I_x - I_y)\omega_x\omega_y = 0$$

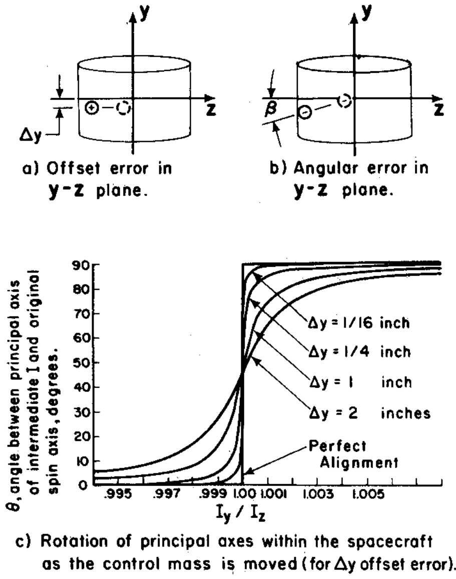

Fig. 2 Effect of alignment errors in spacecraft with the configuration of Fig. 1a, having z_{max} = 60″ and z_{min} = 24″.

The initial conditions (ω_x, ω_y, and ω_z at $t = 0$) are determined by the details of the control mass movement. With no closed form solution available, the inversion action has been studied by numerical integration on a digital computer. The inversion is found to begin slowly, but to accelerate as the tilt angle increases up to $90° - \alpha_0$. It then begins to decelerate, with the maneuver continuing until **y** has turned through the angle $180° - 2\alpha_0$. If the spacecraft were not recaptured at this time by returning the control mass to its normal location, it would go back with a second inversion to its original position, and motion back and forth between the upright and inverted positions would continue indefinitely in the absence of damping. (If damping were applied to the spacecraft in this condition, it would end up spinning stably about **z**. Tilting in 90° steps is therefore feasible; this might be useful for special applications.)

Figure 1 shows different paths that might be used for the control mass movement. In configurations a and b, a single mass is moved in a line nominally coincident with one of the principal axes. In a the mass is moved inward to decrease I_y and I_z while I_x remains approximately the same. In b, the mass is moved outward to increase I_x and I_z without affecting I_y. In c and d, the paths are chosen to essentially double the effect of the control mass. Its movement will simultaneously increase I_z and decrease I_y. With this approach, a pair of masses is preferred to avoid excessively large values of α_0.

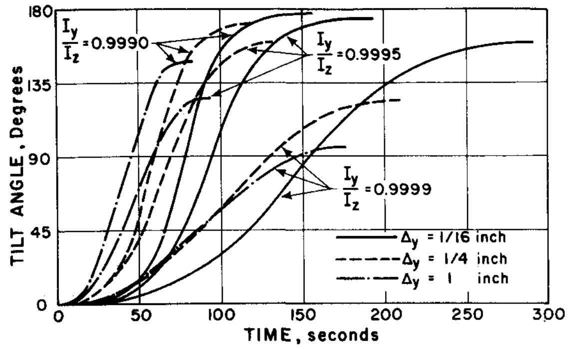

Fig. 3 Inversion action of a spacecraft having the parameters I_z = 76.9 slug-ft², I_x = 73.8 slug-ft², spin rate = 100 rpm, with three different Δy alignment errors (z_{max} = 60″, z_{min} = 24″).

The size of the control mass required depends upon the maximum distance from the center of gravity that it is able to move and, to a lesser degree, upon the minimum distance. In Table 1 are given, for a spacecraft of stated parameters, the size of the control mass required for the four configurations of Fig. 1 with various ranges of travel. It would appear that if the control mass is constrained to remain within the basic satellite structure (within, say, a 26 in. radius of the center of gravity), it must be quite large, so that one should consider the possibility of using a piece of equipment required for other purposes; e.g., a weight of the proper value consisting of a group of electronic units could have its wiring harness built to accommodate the required movement. If the control mass is allowed to extend beyond the basic structure, however, the required size becomes much smaller. There is a point of diminishing return, though, since the weight of a long boom (which should be relatively rigid) could exceed that of the control mass itself.

Another factor to consider when comparing different configurations is the maximum force or torque required to overcome centrifugal force when moving the control mass. These values (Table 1) are reasonable but not insignificant. Force due to Coriolis acceleration should also be considered; this is perpendicular to the path of the movement and proportional to the control mass velocity.

If dimensional tolerances are not close enough, there may be an excessive amount of nutation after recapture, or the maneuver may occur so rapidly that the timing of recapture is very critical. The characteristics of the inversion are highly dependent upon both α_0 and the I_y/I_z ratio; the I_x/I_y ratio being less significant. To obtain reasonable timing, it would appear that I_y/I_z should probably lie between 0.99900 and 0.99990 during the maneuver. The tolerance of the position of the control mass corresponding to these two values (Table 1) is found to be quite large in all cases.

Table 1 Control mass parameters for inverting a spacecraft[a]

Config. (Fig. 1)	Extreme positions of control mass[b]				Req'd control mass, lb	Max. force or torque to move mass	Tolerance on mass position for I_y/I_z between 0.9990 and 0.9999, in.
	z_{min}	z_{max}	y_{min}	y_{max}			
a)	12″	26″	0″	0″	25.6	189 lb	0.53
a)	12″	40″	0″	0″	9.62	109 lb	1.33
a)	24″	60″	0″	0″	4.68	79.7 lb	1.40
b)	0″	0″	12″	26″	26.3	0 while mass	0.25
b)	0″	0″	12″	40″	9.74	is being	0.43
b)	0″	0″	24″	60″	4.70	moved[c]	0.59
c)	12″	26″	0″	26″	11.9 total	20.8 lb each	0.50
d)	28″	38″	28″	38″	10.8 total	37.2 ft-lb each	0.54
d)	28″	48″	28″	48″	4.69 total	38.3 ft-lb each	1.22
d)	28″	60″	28″	60″	2.53 total	36.1 ft-lb each	2.22

[a] Having I_y = 80 slug-ft², I_y/I_z = 1.04 during normal operation, wt = 770 lb, spin rate = 100 rpm.
[b] With respect to principal axes of normal operating condition.
[c] Maximum forces on mass are comparable to those of configuration a during the maneuver (when position of mass is fixed).

α_0 depends upon: 1) alignment of the control mass path with the principal axes, 2) I_y/I_z at the start of the maneuver, and 3) timing of the mass movement. Misalignment data presented here are for the system of Fig. 1a, but the effects of misalignment in the other configurations are essentially the same. With perfect alignment; i.e., with the path of the control mass coincident with the **z** principal axis, the directions of the principal axes remain fixed within the spacecraft as the control mass is moved (the shift in the center of gravity has no significant effect), and θ (the angle between the principal axis of intermediate moment of inertia and the original spin axis) changes abruptly from 90° to 0° as I_y/I_z changes from a value greater than 1.0 to less than 1.0. Figures 2a and 2b show two types of misalignment that will be present to some degree in any real system; their effects are very similar. Figure 2c (from fundamental principal axis equations) shows the effects of various Δy offset errors on θ.

Misalignment in the **x-z** plane, offset or angular, will not in itself affect α_0. Movement of the mass with such misalignment rotates the **x** and **z** principal axes without affecting the direction of y.

Euler's equations are not valid for the period of mass movement. For rigorous computer simulation of this phase, equations have been developed using the method of Grubin.[2] I_i, I_j, I_k are defined as the moments of inertia of the main body about its principal axes **i,j,k** (corresponding, respectively, to **x,y,z** of the total vehicle, but not normally quite aligned with them), and the control mass is assumed to be a point mass. M is the mass of the main body and m that of the control mass. For the system of Fig. 1a, with the path of the control mass confined to the **j-k** plane, and r_k being the instantaneous distance of the mass from the **j** axis and r_j its instantaneous distance from the **k** axis, the equations of motion are

$$\dot{\omega}_i = \frac{-1}{I_i + m'(r_k^2 + r_j^2)} \{[I_j - I_k + m'(r_k^2 - r_j^2)]\omega_j\omega_k + m'[2(r_k\dot{r}_k + r_j\dot{r}_j)\omega_i + r_jr_k(\omega_j^2 - \omega_k^2) + r_k\ddot{r}_j - r_j\ddot{r}_k]\}$$

$$\dot{\omega}_j = \frac{-[(I_k + m'r_j^2)A + m'r_jr_kB]}{I_jI_k + m'(r_j^2I_j + r_k^2I_k)}$$

$$\dot{\omega}_k = \frac{-[m'r_jr_kA + (I_j + m'r_k^2)B]}{I_jI_k + m'(r_j^2I_j + r_k^2I_k)}$$

$$A = (I_k - I_i)\omega_i\omega_k - m'(r_k^2\omega_i\omega_k + r_jr_k\omega_i\omega_j + 2r_k\dot{r}_j\omega_k - 2r_k\dot{r}_k\omega_j)$$

$$B = (I_i - I_j)\omega_i\omega_j + m'(r_j^2\omega_i\omega_j + r_jr_k\omega_i\omega_k - 2r_j\dot{r}_k\omega_j + 2r_j\dot{r}_j\omega_k)$$

$$m' = Mm/(M + m)$$

When the control mass is stopped, these become equivalent to Euler's equations, although not of the same form since **i,j,k** are not in general aligned with **x,y,z**.

Figure 3 shows simulated inversion for various I_y/I_z ratios and Δy offset errors. The total tilt angle is highly dependent upon the alignment error. With $\Delta y = 1$ in. and $I_y/I_z = 0.9990$, for example, the spacecraft tilts 147°, leaving a 33° nutation angle to be damped out. With I_y/I_z the same but $\Delta y = \frac{1}{16}$ in., however, the nutation angle is only 2°. Figure 3 is based on $I_x = 73.8$ slug-ft² during inversion. (With the configuration of Fig. 1a, such a spacecraft would have $I_x = I_z$ during normal operation.) A reduction in I_x would cause the maneuver to occur more rapidly.

For uniformity, Fig. 3 is based on the initial tilt angle α_0 being identical to the angle of rotation of **y** within the spacecraft (i.e., the value of θ in Fig. 2c corresponding to I_y/I_z at the end of the control mass movement). The rigorous simulations of the period of mass movement show that this is quite accurate if that movement occurs within a few seconds. If it takes longer, wobble builds up, causing inversion to occur somewhat more rapidly after the mass is stopped, but even a period of 30 sec does not greatly affect the data of Fig. 3.

A mechanically despun antenna could be allowed to spin up until it was fixed with respect to the basic spacecraft just before the maneuver, then be despun again after inversion. (It must rotate, with respect to the spacecraft, in the opposite direction after inversion.)

The smaller the I_y/I_z ratio in normal operation, the smaller the required control mass. Spin stability decreases as I_y/I_z approaches 1.00, however, so a tradeoff is necessary. The value 1.04 used for developing the data in this paper has proven to be satisfactory for the ATS-3 (Applications Technology Satellite) and it is likely that a smaller value would be practical for many missions.

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Hydrazine Azide as an Additive for Monopropellant Hydrazine

Thomas F. Seamans* and Jack Kahrs†
Thiokol Chemical Corporation, Denville, N. J.

and

George R. Huson‡
COMSAT Laboratories, Clarksburg, Md.

Introduction

Hydrazine is used for various satellite propulsion applications because of its simplicity, its demonstrated near-theoretical performance, and its long-term stability. This Note summarizes results of an investigation of blends of hydrazine and hydrazine azide ($N_2H_4HN_3$, whose acronym is HA) offering significantly better theoretical performance. Based on liquid reactants at 298°K, Fig. 1 shows the standard performance parameters vs the computed fraction of ammonia dissociated. Adiabatic flame temperature decreases sharply as the fraction dissociated increases, and increases as the proportion of HA in the blend increases. Blends of 20–26% HA are particularly attractive because the freezing point is lower and performance is higher than with neat hydrazine.

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* Section Chief, Physics and Combustion Section, Reaction Motors Division; presently with Ionics Inc., Watertown, Mass.

† Research Specialist, Physics and Combustion Section, Reaction Motors Division; presently with Sandvik Conveyor Inc., Fair Lawn, N. J. Member AIAA.

‡ Section Head, Propulsion, Positioning and Orientation Branch. Member AIAA.